Solving problems by searching

Chapter 3

Outline

- Problem-solving agents
- Problem types
- Problem formulation
- Example problems
- Basic search algorithms

Problem-solving agents

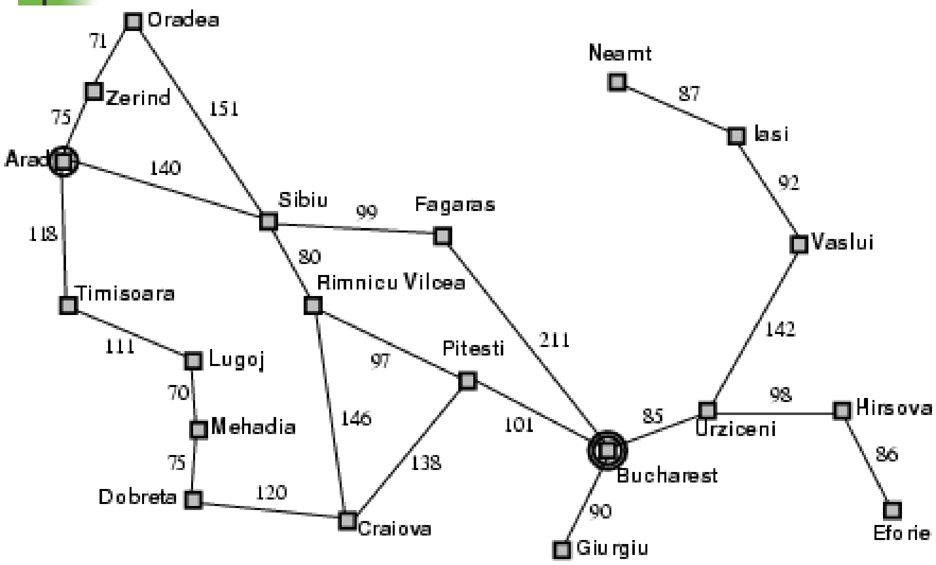
```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
   static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

   state ← UPDATE-STATE(state, percept)
   if seq is empty then do
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH( problem)
   action ← FIRST(seq)
   seq ← REST(seq)
   return action
```


Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest□
- Formulate goal:
  - be in Bucharest□
- Formulate problem:
  - states: various cities
  - actions: drive between cities□
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest□

Example: Romania

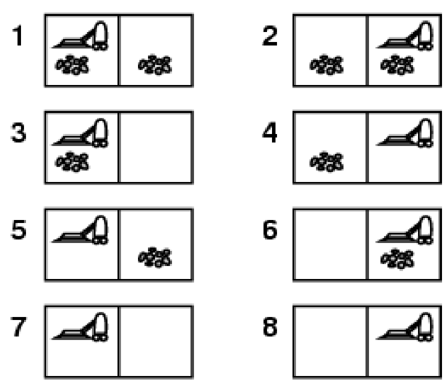


Problem types

- Deterministic, fully observable → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable → sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable → contingency problem
  - percepts provide new information about current state
  - often interleave} search, execution
- Unknown state space → exploration problem

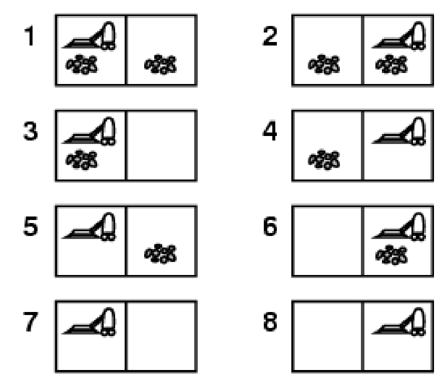
Example: vacuum world

- Single-state, start in #5. Solution?



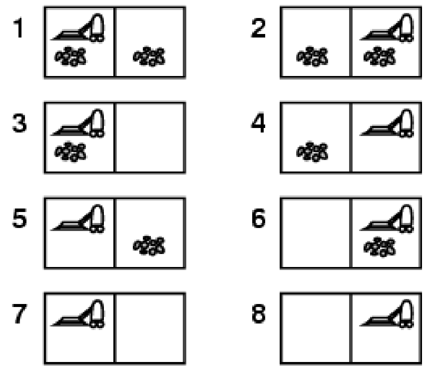
Example: vacuum world

- Single-state, start in #5. Solution? *[Right, Suck]*
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution?



Example: vacuum world

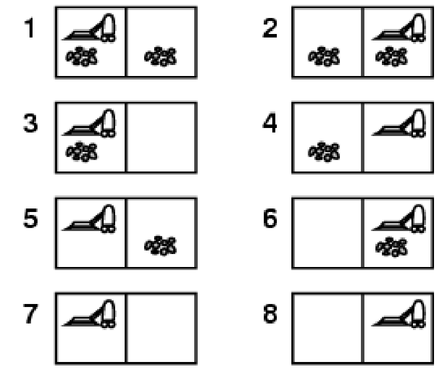
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution? *[Right,Suck,Left,Suck]*□



- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean]*, i.e., start in #5 or #7
    Solution?

Example: vacuum world

- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution? *[Right,Suck,Left,Suck]*□



- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean]*, i.e., start in #5 or #7
    Solution? *[Right, **if** dirt **then** Suck]*

Single-state problem formulation

A problem is defined by four items:□

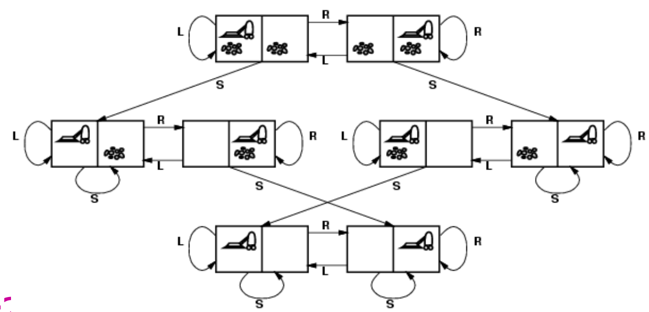
1. initial state e.g., "at Arad"□
2. actions or successor function *S(x)* = set of action–state pairs
   - e.g., *S(Arad) = { <Arad → Zerind, Zerind>, … }*□
3. goal test, can be
   - explicit, e.g., *x* = "at Bucharest"
   - implicit, e.g., *Checkmate(x)*□
4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - *c(x,a,y)* is the step cost, assumed to be ≥ 0□

- A solution is a sequence of actions leading from the initial state to a goal state□

Selecting a state space

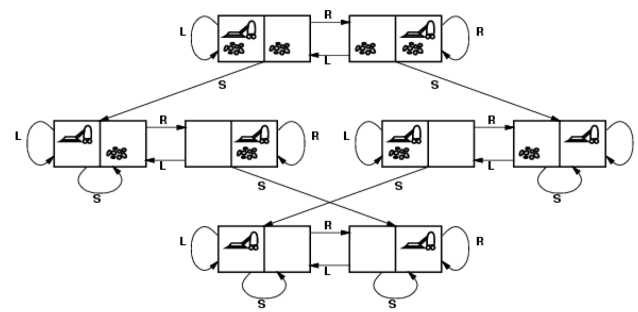
- Real world is absurdly complex
  → state space must be abstracted for problem solving□
- (Abstract) state = set of real states□
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"□
- (Abstract) solution =
  - set of real paths that are solutions in the real world□
- Each abstract action should be "easier" than the original problem□

Vacuum world state space graph



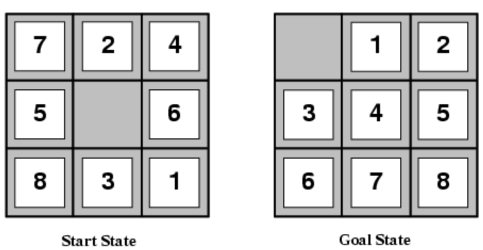
- states?
- actions?
- goal test?
- path cost?□

Vacuum world state space graph



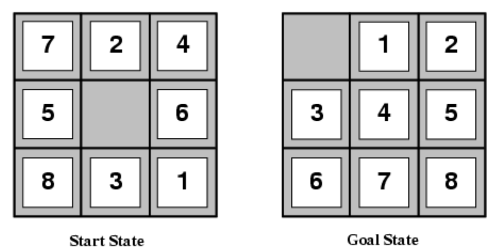
- states? integer dirt and robot location
- actions? *Left*, *Right*, *Suck*
- goal test? no dirt at all locations
- path cost? 1 per action

Example: The 8-puzzle



- states?
- actions?
- goal test?
- path cost?

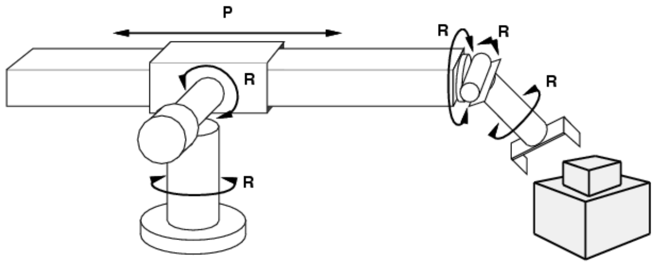
Example: The 8-puzzle



- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move□

[Note: optimal solution of *n*-Puzzle family is NP-hard]□

Example: robotic assembly



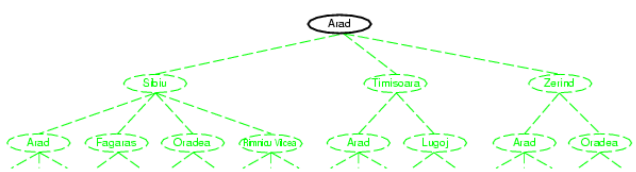
- states?: real-valued coordinates of robot joint angles parts of the object to be assembled□
- actions?: continuous motions of robot joints□
- goal test?: complete assembly□
- path cost?: time to execute□

Tree search algorithms

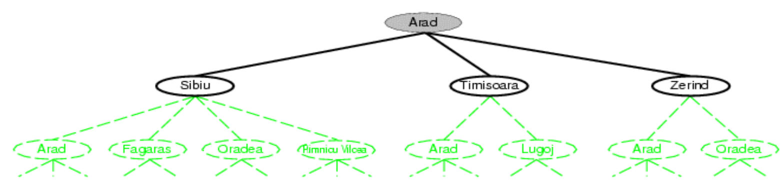
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)□

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
     if there are no candidates for expansion then return failure
     choose a leaf node for expansion according to strategy
     if the node contains a goal state then return the corresponding solution
     else expand the node and add the resulting nodes to the search tree
```

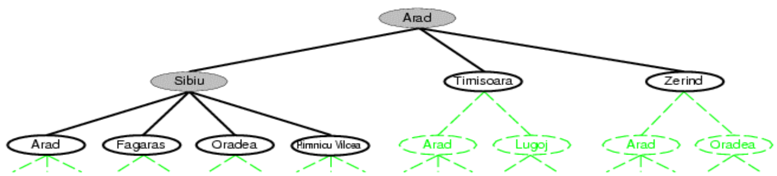

Tree search example



Tree search example



Tree search example



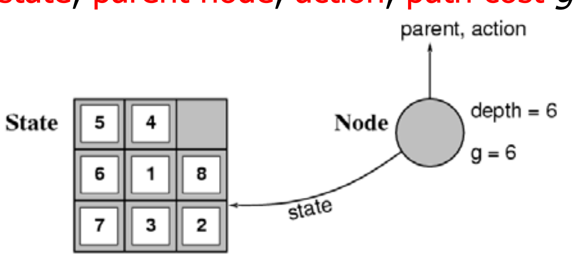
Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← REMOVE-FRONT(fringe)
       if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
       fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
   successors ← the empty set
   for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
       s ← a new NODE
       PARENT-NODE[s] ← node;   ACTION[s] ← action;   STATE[s] ← result
       PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
       DEPTH[s] ← DEPTH[node] + 1
       add s to successors
   return successors
```


Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



- The `Expand` function creates new nodes, filling in the various fields and using the `SuccessorFn` of the problem to create the corresponding states.□

Search strategies

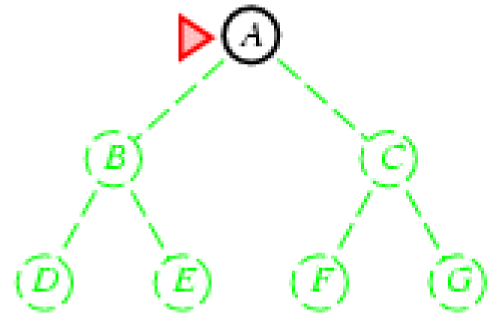
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?□
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)□

Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition□
- Breadth-first search□
- Uniform-cost search□
- Depth-first search□
- Depth-limited search□
- Iterative deepening search□

Breadth-first search

- Expand shallowest unexpanded node□
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end□



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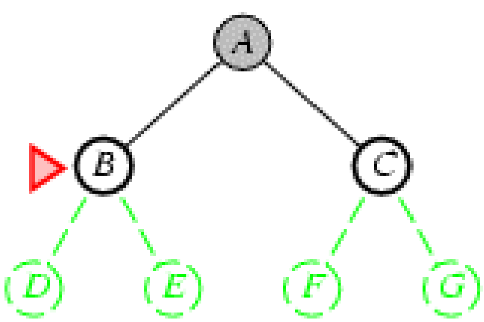

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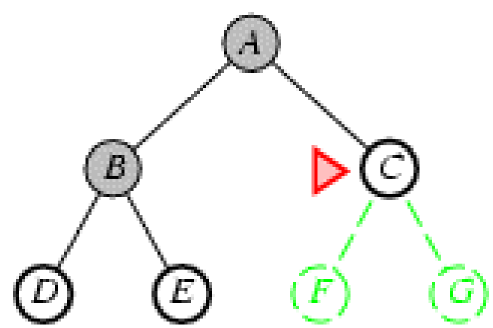

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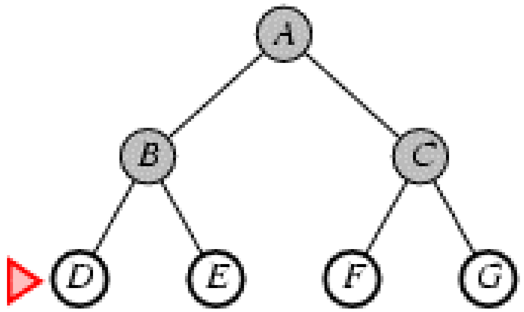


Properties of breadth-first search

- Complete? Yes (if b is finite)□
- Time? $1+b+b^2+b^3+\ldots+b^d + b(b^d-1) = O(b^{d+1})$□
- Space? $O(b^{d+1})$ (keeps every node in memory)□
- Optimal? Yes (if cost = 1 per step)□

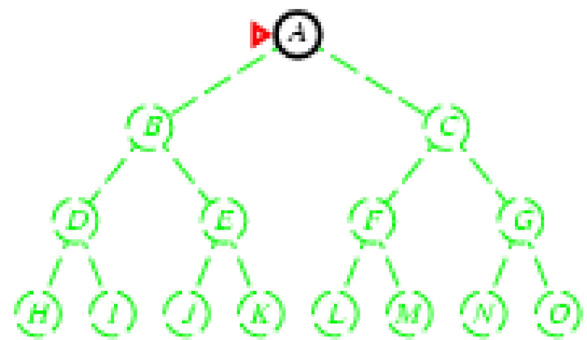
- Space is the bigger problem (more than time)□

Uniform-cost search

- Expand least-cost unexpanded node□
- Implementation:
  - *fringe* = queue ordered by path cost□
- Equivalent to breadth-first if step costs all equal□
- Complete? Yes, if step cost ≥ ε□
- Time? # of nodes with $g \leq$ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$ where C^* is the cost of the optimal solution
- Space? # of nodes with $g \leq$ cost of optimal solution, $O(b^{ceiling(C^*/\varepsilon)})$□
- Optimal? Yes – nodes expanded in increasing order of $g(n)$□

Depth-first search

- Expand deepest unexpanded node□
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front□



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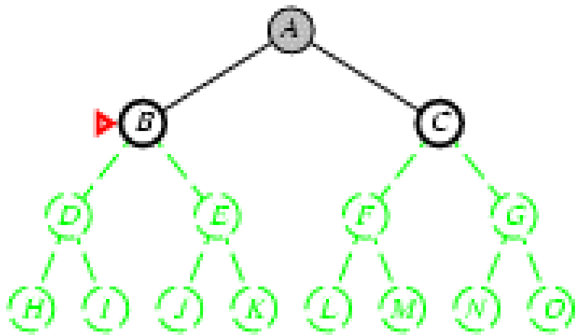

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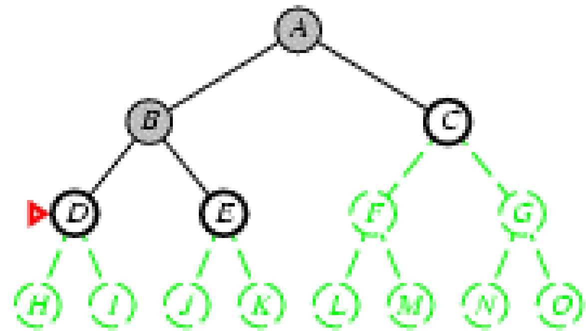

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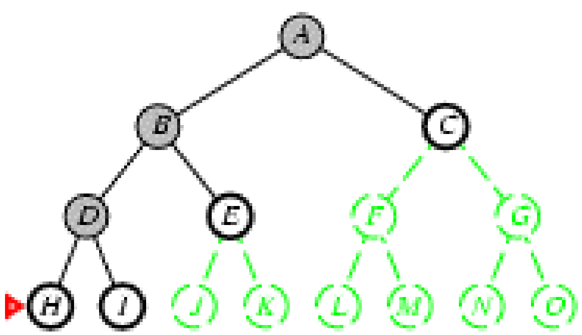

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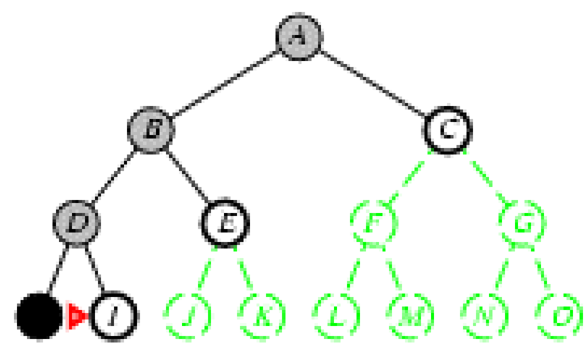

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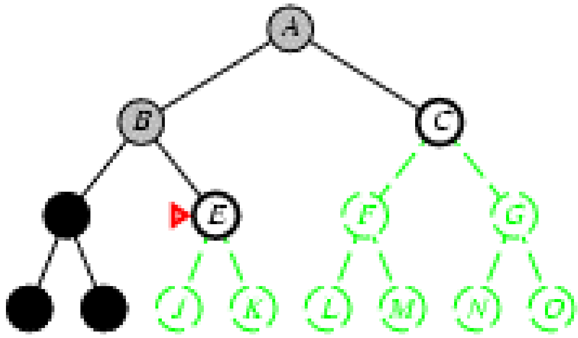

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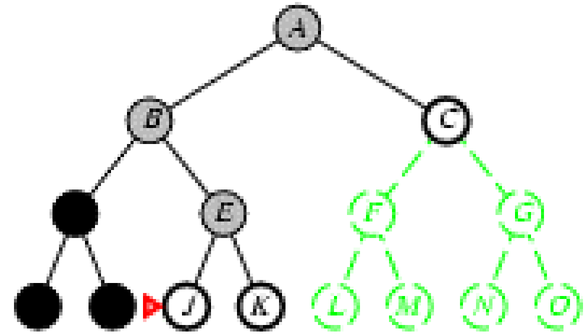

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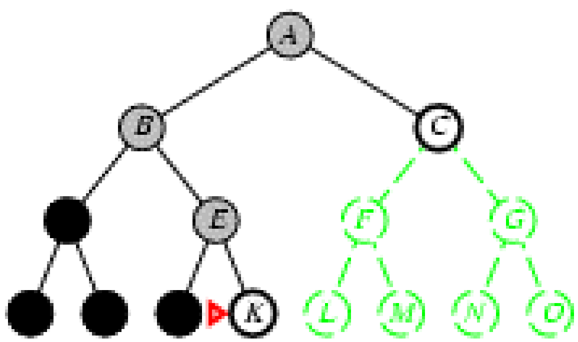

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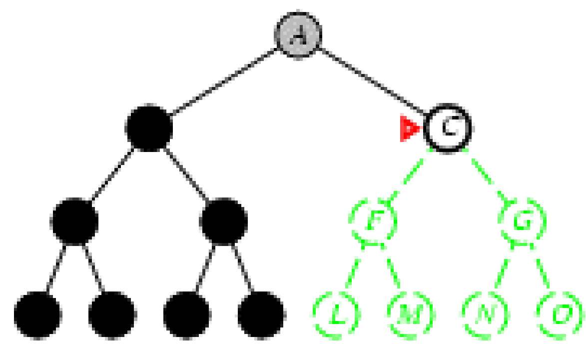

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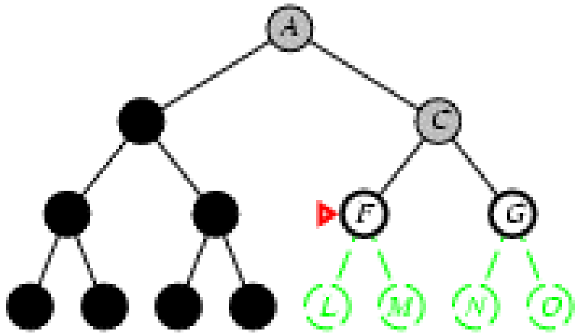

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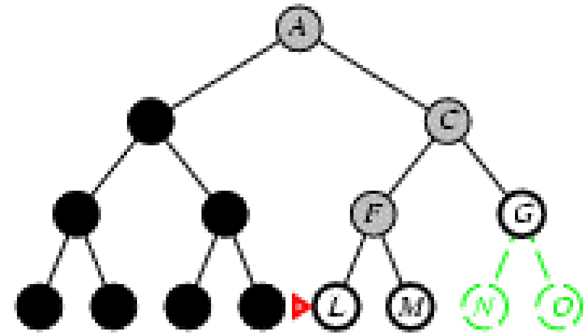

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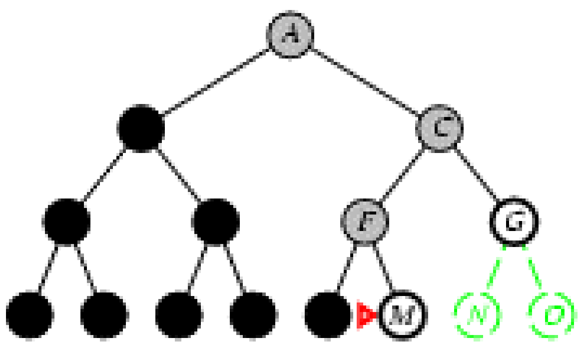


Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path□
    - → complete in finite spaces□
- Time? $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first□
- Space? $O(bm)$, i.e., linear space!□
- Optimal? No□

Depth-limited search

= depth-first search with depth limit *l*,
i.e., nodes at depth *l* have no successors□

- Recursive implementation:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

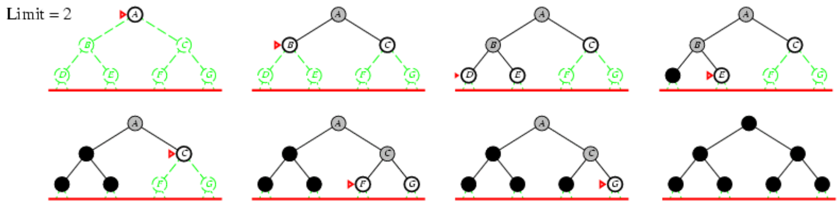

Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or failure
   inputs: problem, a problem
   for depth ← 0 to ∞ do
      result ← DEPTH-LIMITED-SEARCH( problem, depth)
      if result ≠ cutoff then return result
```

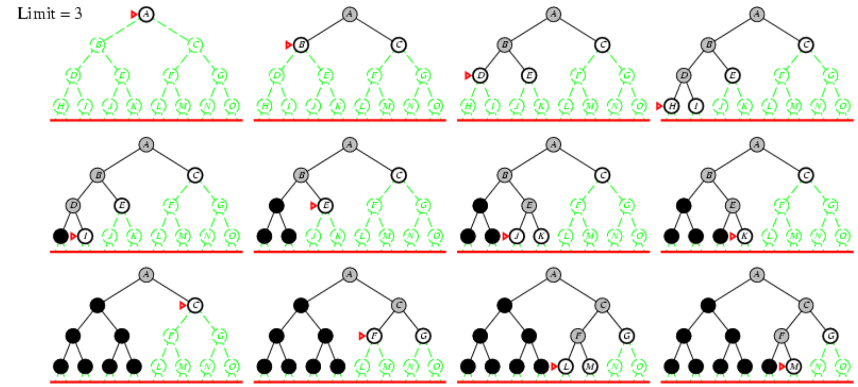

Iterative deepening search *l* =0

Iterative deepening search *l* =1

Iterative deepening search $l = 2$



Iterative deepening search $l = 3$



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth d with branching factor b:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth d with branching factor b:

$$N_{IDS} = (d+1)b^0 + d\ b\text{^}1 + (d-1)b\text{^}2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10$, $d = 5$,□
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$□
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$□
- Overhead = (123,456 - 111,111)/111,111 = 11%

Properties of iterative deepening search

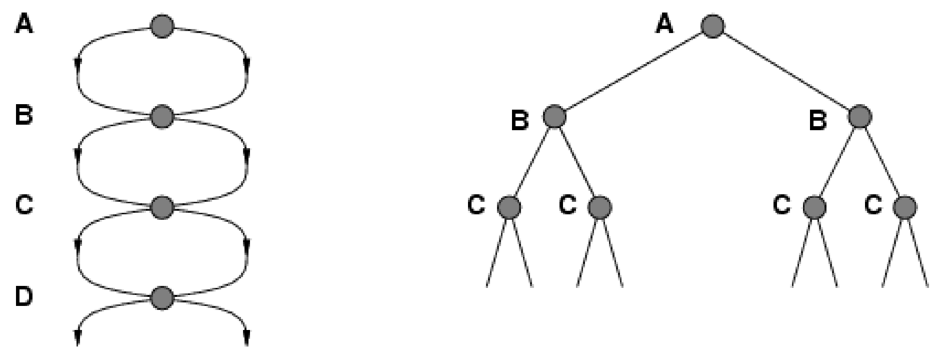
- Complete? Yes□
- Time? $(d+1)b^0 + d\ b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$□
- Space? $O(bd)$□
- Optimal? Yes, if step cost = 1

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!□



Graph search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← REMOVE-FRONT(fringe)
      if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
      if STATE[node] is not in closed then
         add STATE[node] to closed
         fringe ← INSERTALL(EXPAND(node, problem), fringe)
```


Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored□
- Variety of uninformed search strategies□
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms□