

Logic

An interpretation I is a mapping between the elements of the language and the elements of a conceptualization of the world.

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Propositional Logic

- ▶ **Conjunction** $A \wedge B$
- ▶ **Disjunction** $B \vee C$
- ▶ **Implication** $A \rightarrow B$
premise, antecedent A
conclusion, consequent B
- ▶ **Equivalence** $A \leftrightarrow B$
- ▶ **Negation** $\neg B$

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Logic

- ▶ **Syntax**
 $x \geq y$ **Yes!**
 $x y \geq$ **No!**
- ▶ **Semantics**
 - ▶ $x \geq y$ **FALSE** when y is bigger than x
 - ▶ Under what states of affairs or worlds is $x \geq y$ true.

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Logic

- ▶ **Entailment**
 $KB \models \alpha$
- ▶ **Derivability**
 $KB \vdash \alpha$
- $KB \vdash \alpha$
- ▶ **Sound**
- ▶ **Complete**

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Propositional Logic

Semantics

- ▶ **P** "A is on B"
- ▶ $I(P) = \text{True}$ or $I(P) = \text{False}$ for all propositional symbols P.
- ▶ **True** – true
- ▶ **False** – false

Propositional Logic

Syntax

True, False
 P, Q, ... Propositional Symbols
 $\wedge, \vee, \leftrightarrow, \rightarrow, \neg, (,)$
 Sentence \Rightarrow

- True
- False
- Propositional/Symbol
- (Sentence)
- Sentence \wedge Sentence
- Sentence \vee Sentence
- Sentence \rightarrow Sentence
- Sentence \leftrightarrow Sentence
- \neg Sentence

Propositional Logic

Truth Tables

P	Q	$P \rightarrow Q$
true	true	true
false	true	true
true	false	false
false	false	true

P	Q	$P \leftrightarrow Q$
true	true	true
false	true	false
true	false	false
false	false	true

Propositional Logic

Truth Tables

P	Q	$P \wedge Q$
true	true	true
false	true	false
true	false	false
false	false	false

P	Q	$P \vee Q$
true	true	true
false	true	true
true	false	true
false	false	false

Propositional Logic

$$I(P) = T$$

$$I(Q) = F$$

$$I(R) = F$$

$$I(S) = T$$

$$I \models (P \wedge Q) \vee R$$

Propositional Logic

Truth Tables

P	$\neg P$
true	false
false	true

Propositional Logic

Truth Tables

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$	$(\neg P \vee Q) \leftrightarrow (P \rightarrow Q)$
true	true	false	true	true	true
false	true	true	true	true	true
true	false	false	false	false	true
false	false	true	true	true	true

Propositional Logic

Semantics

- ▶ Interpretation
- ▶ Compositionality
- ▶ Valid, Tautology
- ▶ Satisfiable
- ▶ Unsatisfiable

Rules of Inference

And-Elimination

$$\frac{\alpha_1 \wedge \alpha_2 \dots \wedge \alpha_n}{\alpha_i}$$

And-Introduction

$$\frac{\alpha_1, \alpha_2, \dots, \alpha_n}{\alpha_1 \wedge \alpha_2 \dots \wedge \alpha_n}$$

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Rules of Inference

Unit Resolution

$$\frac{\alpha \vee \beta, \neg\beta}{\alpha}$$

Resolution

$$\frac{\alpha \vee \beta, \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

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Propositional Logic

Rules of Inference

$$\alpha \vdash \beta$$

$$\frac{\alpha}{\bar{\beta}}$$

Modus Ponens

$$\frac{\alpha \rightarrow \beta, \alpha}{\beta}$$

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Rules of Inference

Or-Introduction

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \dots \vee \alpha_n}$$

Double Negation Elimination

$$\frac{\neg\neg\alpha}{\alpha}$$

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