Theorem Proving

Resolution Refutation Proofs

$$\Delta \models \alpha$$

$$\Delta, \neg\alpha \not\models$$

Theorem Proving

Notation

$$\text{Clause} \stackrel{\text{def}}{=} \text{Set of Literals}$$

$$\text{Literal} \stackrel{\text{def}}{=} \begin{array}{l}\text{Atomic Sentence}\\ \text{or its negation}\end{array}$$

$\{P, \neg P, Q$

$\{P, \neg P\}$
$\{R\}$
$\{S\}$

Theorem Proving

Example

1. $\{P, Q\}$ given
2. $\{\neg P, R\}$ given
3. $\{Q, R\}$ 1,2

Theorem Proving

Resolution Rule of Inference

$$\frac{\alpha \vee \beta, \neg\beta \vee \delta}{\alpha \ \vee \ \delta}$$

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Set Notation

$$\frac{\begin{array}{l}\Gamma \textit{ with } \alpha \in \Gamma \\ \Delta \textit{ with } \neg\alpha \in \Delta\end{array}}{(\Gamma - \alpha) \cup (\Delta - \neg\alpha)}$$

Theorem Proving

1. $\{P\}$ given
2. $\{\neg P\}$ given
3. $\{\}$ 1,2

Empty Clause: $F, \{\}, \Box$

Conversion to Clause form

1. Eliminate Implications

$$\alpha \rightarrow \beta$$

is replaced by

$$\neg\alpha \vee \beta$$

Conversion to Clause form 2. Move Negations Inwards

Move $\neg$ inwards so that $\neg$ only applies to propositional symbols.

$\neg\neg\beta$ *is replaced by* β

$\neg(\alpha \wedge \beta)$ *is replaced by* $(\neg\alpha \vee \neg\beta)$

$\neg(\alpha \vee \beta)$ *is replaced by* $(\neg\alpha \wedge \neg\beta)$

Conversion to Clause form

3. Put in Conjunctive Normal Form Distribute

$$\alpha \vee (\beta \wedge \delta)$$

becomes

$$(\alpha \vee \beta) \wedge (\alpha \vee \delta)$$

Flatten

$$(\alpha \vee \beta) \vee \delta$$

becomes

$$(\alpha \vee \beta \vee \delta)$$

$$(\alpha \wedge \beta) \wedge \delta$$

becomes

$$(\alpha \wedge \beta \wedge \delta)$$

Conversion to Clause Form

Examples

1. $P \wedge Q \wedge R$

   $$\{P\}, \{Q\}, \{R\}$$

2. $(P \vee Q) \wedge (\neg P \vee Q) \wedge R$

   $$\{P, Q\}, \{\neg P, Q\}, \{R\}$$

Conversion to Clause Form

Examples

3. $\neg((P \vee Q) \wedge \neg Q)$

$$\neg(P \vee Q) \vee \neg\neg Q$$
$$(\neg P \wedge \neg Q) \vee Q$$
$$(\neg P \vee Q) \wedge (\neg Q \vee Q)$$
$$\{\neg P, Q\}, \{\neg Q, Q\}$$

Theorem Proving

The Procedure

$$\Delta \stackrel{?}{\models} \alpha$$

$$\Delta \cup \neg\alpha \stackrel{?}{\not\models}$$

1. Clausify $(\Delta \cup \neg\alpha)$
2. Repeatedly try to produce $\Box$ by performing resolution and adding the resolvent to the set of clauses.

Theorem Proving

Examples

1. $P \vee Q$
2. $\neg P \vee Q$
3. $P \vee \neg Q$
4. $\neg P \vee \neg Q$
5. Q 1,2
6. $\neg Q$ 3,4
7. $\square$ 5,6

Theorem Proving

Examples

1. $\neg P \vee Q$
2. $\neg Q$
3. P
4. $\neg P$ 1,2
5. $\Box$ 3,4