Blocks World Example

```
  a
  b
  c
d  e
```

Domain and Relations

- **Domain**: $D = \{a, b, c, d, e\}$
- **Relations**:
  - $\{(a, b), (b, c), (d, e)\}$
  - $\{(a, b), (b, c), (a, c), (d, e)\}$
  - $\{(c, e)\}$
  - $\{(a, d)\}$

Functions

- **Functions**:
  - $\{(b) \rightarrow a, (c) \rightarrow b, (e) \rightarrow d\}$

Non-Logical Symbols

**Predicate Symbols**
- ABOVE
- TABLE
- CLEAR
- ON

**Function Symbols**
- BLOCKA
- BLOCKB
- BLOCKC
- BLOCKD
- BLOCKE
Interpretation Mapping

\begin{align*}
\mathcal{I}[\text{ON}] &= \{(a, b), (b, c), (d, e)\} \\
\mathcal{I}[\text{ABOVE}] &= \{(a, b), (b, c), (a, c), (d, e)\} \\
\mathcal{I}[\text{BLOCKA}] &= a \\
\mathcal{I}[\text{BLOCKB}] &= b \\
\mathcal{I}[\text{BLOCKC}] &= c \\
\mathcal{I}[\text{BLOCKD}] &= d \\
\mathcal{I}[\text{BLOCKE}] &= e \\
\mathcal{I}[\text{HAT}] &= \{(b) \rightarrow a, (c) \rightarrow b, (e) \rightarrow d\} \\
\mathcal{I}[\text{TABLE}] &= \{c, e\} \\
\mathcal{I}[\text{CLEAR}] &= \{a, d\}
\end{align*}

Another Interpretation (Non-Intended)

\begin{align*}
\mathcal{I}[\text{ABOVE}] &= \{(b, a), (c, b), (e, d)\} \\
\mathcal{I}[\text{ON}] &= \{(b, a), (c, b), (c, a), (e, d)\} \\
\mathcal{I}[\text{BLOCKA}] &= a \\
\mathcal{I}[\text{BLOCKB}] &= c \\
\mathcal{I}[\text{BLOCKC}] &= b \\
\mathcal{I}[\text{BLOCKD}] &= b \\
\mathcal{I}[\text{BLOCKE}] &= e \\
\mathcal{I}[\text{HAT}] &= \{(b) \rightarrow a, (c) \rightarrow b, (e) \rightarrow d\} \\
\mathcal{I}[\text{CLEAR}] &= \{c, e\} \\
\mathcal{I}[\text{TABLE}] &= \{a, d\}
\end{align*}

Example

Does the following formula hold in the intended interpretation?

\[ \exists \models \text{ABOVE}(\text{BLOCKA}, \text{BLOCKB}) \]

Example

Does the following formula hold in the intended interpretation?

\[ \exists \models \text{ABOVE}(\text{HAT} (\text{BLOCKB}), \text{BLOCKB}) \]
Example

Does the following formula hold in the intended interpretation?

\[ \exists \models \text{Above}(\text{blocka, blockb}) \land \text{Above}(\text{blocka, blockc}) \]

Variables

ON(x, y) → ABOVE(x, y)

Variable Assignment \( \mu \)

\[ \mu(x) = a \]
\[ \mu(y) = a \]
\[ \mu(z) = b \]

Example

\[ \exists \mu' \models \forall x. \text{ON}(x, \text{blocka}) \]

Example

\[ \forall x, y \text{ON}(x, y) \rightarrow \text{ABOVE}(x, y) \]
Example

\exists y \text{Above}(y, \text{BLOCKC})

Denotation

variable assignment \(\mu\) over \(D\)

1. If \(x\) is a variable then \([x]_\exists,\mu = \mu[x]\)

2. If \(t_1, \ldots, t_n\) are terms, and \(F\) is a function symbol of arity \(n\) then

\([F(t_1, \ldots, t_n)]_\exists,\mu = \mathcal{I}(F)([t_1]_\exists,\mu, \ldots, [t_n]_\exists,\mu)\)

Satisfaction

1. \(\exists, \mu \models P(t_1 \ldots t_n)\) iff \((d_1, \ldots, d_n) \in \mathcal{I}(P)\),
    where \(d_i = [t_i]_\exists,\mu\)

2. \(\exists, \mu \models t_1 = t_2\) iff \([t_1]_\exists,\mu\) and \([t_2]_\exists,\mu\) and are
    the same element of \(D\).

3. \(\exists, \mu \models \neg \alpha\) iff it is not the case that \(\exists, \mu \models \alpha\)

4. \(\exists, \mu \models (\alpha \land \beta)\) iff \(\exists, \mu \models \alpha\) and
    \(\exists, \mu \models \beta\).

5. \(\exists, \mu \models (\alpha \lor \beta)\) iff \(\exists, \mu \models \alpha\) or
    \(\exists, \mu \models \beta\).

6. \(\exists, \mu \models \exists x. \alpha\) iff \(\exists, \mu' \models \alpha\) for some variable
    assignment \(\mu'\) that differs from \(\mu\) on at most
    \(x\).

7. \(\exists, \mu \models \forall x. \alpha\) iff \(\exists, \mu' \models \alpha\) for every variable
    assignment \(\mu'\) that differs from \(\mu\) on at most
    \(x\).

Another Example

\(D = \{\text{GeorgeBush}, \text{ArnoldSchwartzenegger},\)
\(\text{BillClinton}, \text{MargaretThatcher},\)
\(\text{Aristotle}, \text{Mozart}\}\)
Non-Logical Symbols

**Predicate Symbols**

- H
- M
- LOVES

**Function Symbols**

- ARNIE
- BILL
- GEORGE
- MAGGIE
- ARI
- WOLFGANG

Interpretation Mapping

\[ \mathcal{I}[\text{LOVES}] = \{ (\text{ArnoldSchwartzenegger, ArnoldSchwartzenegger}), (\text{GeorgeBush, MargaretThatcher}), (\text{MargaretThatcher, GeorgeBush}), (\text{Mozart, BillClinton}), (\text{BillClinton, Aristotle}), (\text{Aristotle, Mozart}) \} \]

\[ \mathcal{I}[\text{ARNIE}] = \text{ArnoldSchwartzenegger} \]
\[ \mathcal{I}[\text{BILL}] = \text{BillClinton} \]
\[ \mathcal{I}[\text{GEORGE}] = \text{GeorgeBush} \]
\[ \mathcal{I}[\text{MAGGIE}] = \text{MargaretThatcher} \]
\[ \mathcal{I}[\text{WOLFGANG}] = \text{Mozart} \]
\[ \mathcal{I}[\text{ARI}] = \text{Aristotle} \]

Interpretation Mapping (cont)

\[ \mathcal{I}(\text{H}) = \{ \text{GeorgeBush, ArnoldSchwartzenegger}, \text{BillClinton, MargaretThatcher}, \text{Aristotle, Mozart} \} \]
\[ \mathcal{I}(\text{M}) = \{ \text{GeorgeBush, MargaretThatcher} \} \]

Statements to Check

\[ \text{H(MAGGIE)} \]
\[ \text{M(WOLFGANG)} \]
\[ \text{L(WOLFGANG, ARI)} \]
\[ \text{L(ARNIE, ARNIE)} \]

\[ \forall x \text{H}(x) \]
\[ \exists x \text{M}(x) \]
Statements to Check

\[ M(\text{arnie}) \rightarrow \exists y \, L(\text{george}, y) \]

\[ M(\text{maggie}) \rightarrow \exists y \, L(\text{george}, y) \]

Statements to Check

\[ \exists x \, H(x) \land M(x) \]