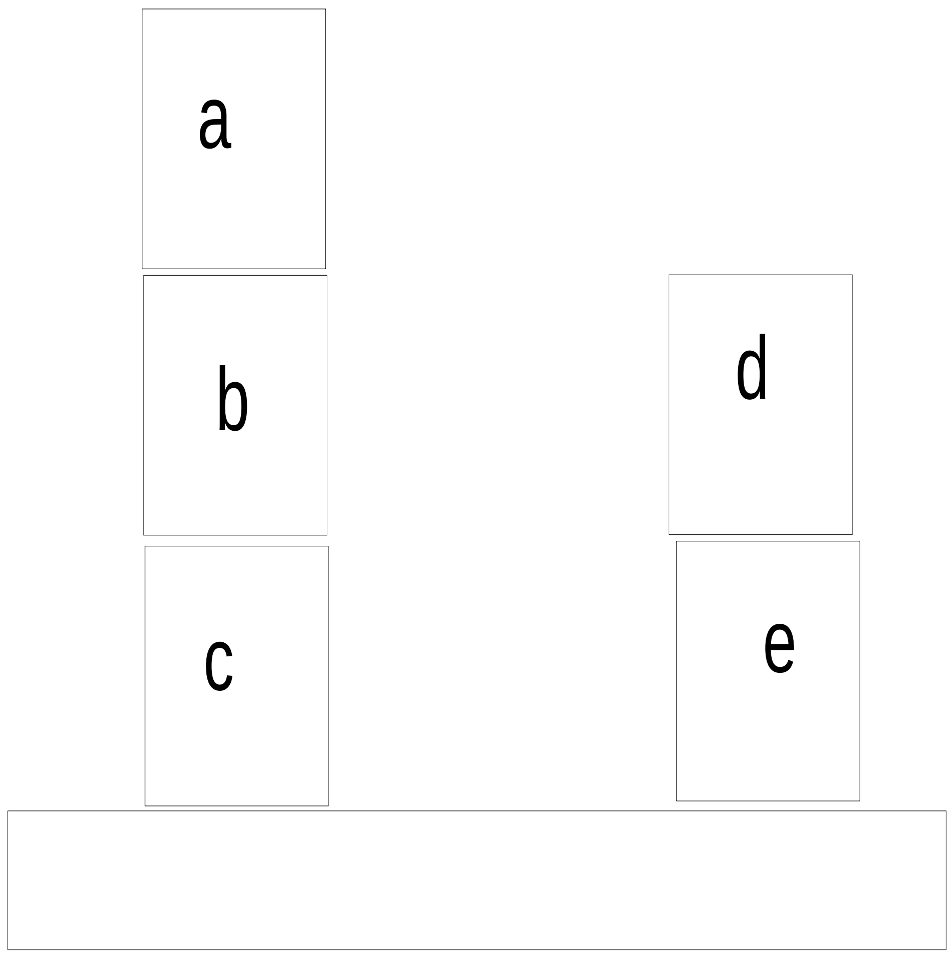
# Blocks World Example



# Domain and Relations

**domain**

$$\mathcal{D} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}\}$$

**relations**

$$\{\langle \mathrm{a}, \mathrm{b}\rangle, \langle \mathrm{b}, \mathrm{c}\rangle, \langle \mathrm{d}, \mathrm{e}\rangle\}$$

$$\{\langle \mathrm{a}, \mathrm{b}\rangle, \langle \mathrm{b}, \mathrm{c}\rangle, \langle \mathrm{a}, \mathrm{c}\rangle, \langle \mathrm{d}, \mathrm{e}\rangle\}$$

$$\{\mathrm{c}, \mathrm{e}\}$$

$$\{\mathrm{a}, \mathrm{d}\}$$

# Functions

## functions

$$\{\langle \mathrm{b} \rangle \to \mathrm{a}, \langle \mathrm{c} \rangle \to \mathrm{b}, \langle \mathrm{e} \rangle \to \mathrm{d}\}$$

# Non-Logical Symbols

## Predicate Symbols

ABOVE

TABLE

CLEAR

ON

## Function Symbols

BLOCKA

BLOCKB

BLOCKC

BLOCKD

BLOCKE

# Interpretation Mapping

$$\mathcal{I}[\text{O}\textsc{n}] = \{\langle \text{a}, \text{b} \rangle, \langle \text{b}, \text{c} \rangle, \langle \text{d}, \text{e} \rangle\}$$

$$\mathcal{I}[\text{A}\textsc{bove}] = \{\langle \text{a}, \text{b} \rangle, \langle \text{b}, \text{c} \rangle, \langle \text{a}, \text{c} \rangle, \langle \text{d}, \text{e} \rangle\}$$

$$\mathcal{I}[\textsc{blocka}] = \text{a}$$

$$\mathcal{I}[\textsc{blockb}] = \text{b}$$

$$\mathcal{I}[\textsc{blockc}] = \text{c}$$

$$\mathcal{I}[\textsc{blockd}] = \text{d}$$

$$\mathcal{I}[\textsc{blocke}] = \text{e}$$

$$\mathcal{I}[\textsc{hat}] = \{\langle \text{b} \rangle \rightarrow \text{a}, \langle \text{c} \rangle \rightarrow \text{b}, \langle \text{e} \rangle \rightarrow \text{d}\}$$

$$\mathcal{I}[\text{T}\textsc{able}] = \{\text{c}, \text{e}\}$$

$$\mathcal{I}[\text{C}\textsc{lear}] = \{\text{a}, \text{d}\}$$

# Another Interpretation (Non-Intended)

$$\mathcal{I}[\textsc{Above}] = \{\langle \mathrm{b}, \mathrm{a}\rangle, \langle \mathrm{c}, \mathrm{b}\rangle, \langle \mathrm{e}, \mathrm{d}\rangle\}$$

$$\mathcal{I}[\textsc{On}] = \{\langle \mathrm{b}, \mathrm{a}\rangle, \langle \mathrm{c}, \mathrm{b}\rangle, \langle \mathrm{c}, \mathrm{a}\rangle, \langle \mathrm{e}, \mathrm{d}\rangle\}$$

$$\mathcal{I}[\textsc{blocka}] = \mathrm{a}$$

$$\mathcal{I}[\textsc{blockb}] = \mathrm{c}$$

$$\mathcal{I}[\textsc{blockc}] = \mathrm{b}$$

$$\mathcal{I}[\textsc{blockd}] = \mathrm{d}$$

$$\mathcal{I}[\textsc{blocke}] = \mathrm{e}$$

$$\mathcal{I}[\textsc{hat}] = \{\langle \mathrm{b}\rangle \rightarrow \mathrm{a}, \langle \mathrm{c}\rangle \rightarrow \mathrm{b}, \langle \mathrm{e}\rangle \rightarrow \mathrm{d}\}$$

$$\mathcal{I}[\textsc{Clear}] = \{\mathrm{c}, \mathrm{e}\}$$

$$\mathcal{I}[\textsc{Table}] = \{\mathrm{a}, \mathrm{d}\}$$

# Example

Does the following formula hold in the intended interpretation?

$$\Im \models \textsc{Above}(\textsc{blocka}, \textsc{blockb})$$

# Example

Does the following formula hold in the intended interpretation?

$$\Im \models \textsc{Above}(\textsc{hat}(\textsc{blockb}), \textsc{blockb})$$

# Example

Does the following formula hold in the intended interpretation?

$\Im \models \textsc{Above}(\textsc{blocka}, \textsc{blockb}) \wedge \textsc{Above}(\textsc{blocka}, \textsc{block}$

# Variables

$$\textsc{On}(x, y) \rightarrow \textsc{Above}(x, y)$$

Variable Assignment $\mu$

$$\mu(x) = \mathrm{a}$$

$$\mu(y) = \mathrm{a}$$

$$\mu(z) = \mathrm{b}$$

# Example

$$\Im\,\mu' \quad \models \quad \forall x.\textsc{On}(x, \textsc{blocka})$$

# Example

$$\forall x, y \textsc{On}(x, y) \rightarrow \textsc{Above}(x, y)$$

# Example

$$\exists y \textsc{Above}(y, \textsc{blockc})$$

# Denotation

variable assignment $\mu$ over $\mathcal{D}$

1. If $x$ is a variable then $[\![x]\!]_{\Im,\mu} = \mu[x]$
2. If $t_1, \ldots, t_n$ are terms, and F is a function symbol of arity $n$ then

   $$[\![\mathrm{F}(t_1, \ldots t_n)]\!]_{\Im,\mu} = \mathcal{I}(\mathrm{F})([\![t_i]\!]_{\Im,\mu}, \ldots, [\![t_i]\!]_{\Im,\mu})$$

# Satisfaction

1. $\Im, \mu \models P(t_1 \ldots t_n)$ iff $\langle d_1, \ldots, d_n \rangle \in \mathcal{I}(\mathrm{P})$, where $d_i = [\![t_i]\!]_{\Im,\mu}$
2. $\Im, \mu \models t_1 = t_2$ iff $[\![t_1]\!]_{\Im,\mu}$ and $[\![t_2]\!]_{\Im,\mu}$ and are the same element of D.
3. $\Im, \mu \models \neg\alpha$ iff it is not the case that $\Im, \mu \models \alpha$
4. $\Im, \mu \models (\alpha \wedge \beta)$ iff $\Im, \mu \models \alpha$ and $\Im, \mu \models \beta$.
5. $\Im, \mu \models (\alpha \vee \beta)$ iff $\Im, \mu \models \alpha$ or $\Im, \mu \models \beta$.
6. $\Im, \mu \models \exists x.\alpha$ iff $\Im, \mu' \models \alpha$ for some variable assignment $\mu'$ that differs from $\mu$ on at most $x$.
7. $\Im, \mu \models \forall x.\alpha$ iff $\Im, \mu' \models \alpha$ for every variable assignment $\mu'$ that differrs from $\mu$ on at most $x$.

# Another Example

$$\mathcal{D} =$$
$$\{\text{GeorgeBush, ArnoldSchwartzenegger,}$$
$$\text{BillClinton, MargaretThatcher,}$$
$$\text{Aristotle, Mozart}\}$$

# Non-Logical Symbols

## Predicate Symbols

H

M

LOVES

## Function Symbols

ARNIE

BILL

GEORGE

MAGGIE

ARI

WOLFGANG

# Interpretation Mapping

$$\mathcal{I}[\text{Loves}] = \\ \{\langle \text{ArnoldSchwartzenegger}, \text{ArnoldSchwartzenegger} \rangle \\ \langle \text{GeorgeBush}, \text{MargaretThatcher} \rangle, \\ \langle \text{MargaretThatcher}, \text{GeorgeBush} \rangle, \\ \langle \text{Mozart}, \text{BillClinton} \rangle, \\ \langle \text{BillClinton}, \text{Aristotle} \rangle, \\ \langle \text{Aristotle}, \text{Mozart} \rangle\}$$

$$\mathcal{I}[\text{ARNIE}] = \text{ArnoldSchwartzenegger}$$

$$\mathcal{I}[\text{BILL}] = \text{BillClinton}$$

$$\mathcal{I}[\text{GEORGE}] = \text{GeorgeBush}$$

$$\mathcal{I}[\text{MAGGIE}] = \text{MargaretThatcher}$$

$$\mathcal{I}[\text{WOLFGANG}] = \text{Mozart}$$

$$\mathcal{I}[\text{ARI}] = \text{Aristotle}$$

# Interpretation Mapping(cont)

$\mathcal{I}$(H) =
{GeorgeBush, ArnoldSchwartzenegger,
BillClinton, MargaretThatcher,
Aristotle, Mozart}

$\mathcal{I}$(M) =
{GeorgeBush, MargaretThatcher}

# Statements to Check

$$\mathrm{H}(\textsc{maggie})$$

$$\mathrm{M}(\textsc{wolfgang})$$

$$\mathrm{L}(\textsc{wolfgang}, \textsc{ari})$$

$$\mathrm{L}(\textsc{arnie}, \textsc{arnie})$$

$$\forall x \mathrm{H}(x)$$

$$\exists x \mathrm{M}(x)$$

# Statements to Check

$$\mathrm{M}(\textsc{arnie}) \rightarrow \exists y \mathrm{L}(\textsc{george}, y)$$

$$\mathrm{M}(\textsc{maggie}) \rightarrow \exists y \mathrm{L}(\textsc{george}, y)$$

# Statements to Check

$$\exists x \mathrm{H}(x) \wedge \mathrm{M}(x)$$