An interpretation $\mathcal{I}$ for DL is a pair

$$\langle D, I \rangle$$

where $D$ is any set of objects called the domain of the interpretation

and

$I$ is a mapping called the interpretation mapping from the non-logical symbols of DL to elements and relations over $D$. 
Semantics (cont)

where

1. for every constant symbol $c$, $\mathcal{I}[c] \in \mathcal{D}$;

2. for every atomic concept $a$, $\mathcal{I}[a] \subseteq \mathcal{D}$;

3. for every role symbol $r$, $\mathcal{I}[c] \subseteq \mathcal{D} \times \mathcal{D}$;
Extending I

- $I[\text{thing}]$
- $I[\text{ALL } r d]$
- $I[\text{EXISTS } n r]$
- $I[\text{FILLS } r c]$
- $I[\text{AND } d_1 \ldots d_n]$
Truth in an Interpretation

Given an interpretation $\mathcal{I}$, we say that $\alpha$ is true in $\mathcal{I}$, or $\mathcal{I} \models \alpha$ according to the following rules.

1. $\mathcal{I} \models (c \rightarrow d)$ iff $\mathcal{I}[c] \in \mathcal{I}[d]$;
2. $\mathcal{I} \models (d \sqsubseteq d')$ iff $\mathcal{I}[d] \subseteq \mathcal{I}[d']$;
3. $\mathcal{I} \models (d \equiv d')$ iff $\mathcal{I}[d] = \mathcal{I}[d']$;

Assuming that $d$ and $d'$ are concepts, and $c$ is a constant.
Entailment

\[ S \models \alpha \]

iff for every \( \mathcal{I} \)

if \( \mathcal{I} \models S \)

then \( \mathcal{I} \models \alpha \)
Examples

\( \text{AND Doctor Female} \sqsubseteq \text{Doctor} \) \)

\( \text{john} \rightarrow \text{Thing} \) \)

\( (\text{Surgeon} \sqsubseteq \text{Doctor}) \) \)

\( KB \models [(\text{AND Doctor Female} \sqsubseteq \text{Doctor})] \) \)

\( (\text{Surgeon} \equiv [\text{AND Doctor [FILLS : Specialty surgery]}) \)
Computing Entailments

We want to be able to determine if $KB \models \alpha$, for sentences $\alpha$ of the form:

$$(c \rightarrow d)$$

$$(d \subseteq e)$$
Simplifying the Knowledge Base

- Remove sentences of the form

  \[(c \rightarrow d)\]

- Left Hand side of \(\subseteq\) and \(\models\) sentences must be atomic concepts other than \textit{Thing} and each atom appears on the left hand side only once.

- Assume \(\subseteq\) and \(\models\) sentences are acyclic.
Normalization

1. expand definitions:

(Surgeon = [AND Doctor [FILLS : Specialty surgery]])

[AND ... Surgeon ...]

expands to

[AND ... [AND Doctor [FILLS : Specialty surgery]] ...]

2. flatten the AND operators:

AND ... [AND $d_1$ ... $d_n$]...

can be simplified to:

AND ... $d_1$ ... $d_n$ ...]
3. combine the \textsc{All} operators:

\[
\text{AND} \ldots [\textsc{All} r d_1] \ldots [\textsc{All} r d_2] \ldots
\]

can be simplified to:

\[
\text{AND} \ldots [\textsc{All} r [\textsc{And} d_1 d_2]] \ldots
\]

4. combine \textsc{Exists} operators:

\[
\text{AND} \ldots [\textsc{Exists} n_1 r] \ldots [\textsc{Exists} n_2 r] \ldots
\]

can be simplified to

\[
\text{AND} \ldots [\textsc{Exists} n r] \ldots
\]

where \( n \) is the maximum of \( n_1 \) and \( n_2 \).
5. deal with Thing:
   Remove vacuous concepts as arguments to AND

   \[\text{ALL } r \text{ Thing}\]

6. remove redundant expressions:
The Final Result

\[ \text{AND } a_1 \ldots a_m \]
\[ \text{[FILLS } r_1 c_1] \ldots [\text{FILLS } r_{m'} c_{m'}] \]
\[ \text{[EXISTS } n_1 s_1] \ldots [\text{EXISTS } n_{m''} s_{m''}] \]
\[ \text{[ALL } t_1 e_1] \ldots [\text{ALL } t_{m'''} e_{m'''}] \]
Structure Mapping

\[ KB \models (d \subseteq e) \]

**IDEA:** For \( d \) to be subsumed by \( e \), the normalized \( d \) must account for each component of the normalized \( e \) in some way.
Structure Mapping Procedure

**Input:** Two normalized concepts $d$ and $e$ where $d$ is of the form $[\text{AND} \ d_1 \ldots d_m]$ and $e$ is of the form $[\text{AND} \ d_1 \ldots d_m]$

**Output** yes or no, according to whether $KB \models (d \sqsubseteq e)$

Return yes iff for each component $e_j$, there exists a component $d_i$ such that $d_i$ matches $e_j$ as follows:

1. if $e_j$ is an atomic concept, then either $d_i$ is identical to $e_j$, or there is a sentence of the form $(d_i \subset d')$ in the KB, where recursively some component of $d'$ matches $e_j$;
2. if $e_j$ is of the form $[\text{FILLS} \ r \ c]$, then $d_i$ must be identical to it;
3. if $e_j$ is of the form $[\text{EXISTS} \ n \ r]$, then the corresponding $d_i$ must be of the form $[\text{EXISTS} \ n' \ r]$, for some $n' \geq n$; if $n = 1$, then $d_i$ may be of the form $[\text{FILLS} \ r \ c]$;
4. if $e_j$ is of the form $[\text{ALL} \ r \ e']$, then $d_i$ must be of the form $[\text{ALL} \ r \ d']$, where recursively $d'$ is subsumed by $e'$.
Computing Satisfaction

\[ KB \models (c \rightarrow e) \]

iff

\[ KB \models (d \subseteq e) \]

where \( d \) is the AND of every concept \( d_i \) such that \((c \rightarrow d_i)\) is in the KB
Taxonomies and Classification

given some query concept, \( q \), find all \( c \) in the KB such that

\[ KB \models (c \rightarrow q) \]

given a constant \( c \), find all the atomic concepts \( a \) such that

\[ KB \models (c \rightarrow a) \]

Partial Order, Classification
Consider adding a sentence \((a \sqsubseteq d)\) to a taxonomy.

1. First calculate \(S\), the most specific subsumers of \(d\)

2. Next calculate \(G\), the most general subsumees of \(d\).

3. If there is a concept \(a'\) in \(S \cap G\), then the concept is already present.

4. Otherwise, insert \(a\).
1. Computing most specific subsumers.

2. Computing the most general subsumees.
Classification (cont)

1. Answering Questions
2. Taxonomies and Frame Hierarchies
3. Inheritance