Semantics

An interpretation $\Im$ for DL is a pair

$$\langle \mathcal{D}, \mathcal{I} \rangle$$

where D is any set of objects called the *domain* of the interpretation

and

I is a mapping called the *interpretation mapping* from the non-logical symbols of DL to elements and relations over D.

Semantics (cont)

where

1. for every constant symbol c, $\mathcal{I}[c] \in \mathcal{D}$;
2. for every atomic concept a, $\mathcal{I}[a] \subseteq \mathcal{D}$;
3. for every role symbol r, $\mathcal{I}[c] \subseteq \mathcal{D} \times \mathcal{D}$;

Extending I

- $\mathcal{I}[\texttt{thing}]$
- $\mathcal{I}[\textbf{ALL}\ r\ d]$
- $\mathcal{I}[\textbf{EXISTS}\ n\ r]$
- $\mathcal{I}[\textbf{FILLS}\ r\ c]$
- $\mathcal{I}[\textbf{AND} d_1 \ldots d_n]$

Truth in an Interpretation

Given an interpretation $\Im$, we say that α is true in $\Im$, or $\Im \models \alpha$ according to the following rules.

1. $\Im \models (c \rightarrow d)$ iff $\mathcal{I}[c] \in \mathcal{I}[d]$;
2. $\Im \models (d \sqsubseteq d')$ iff $\mathcal{I}[d] \subseteq \mathcal{I}[d']$;
3. $\Im \models (d \doteq d')$ iff $\mathcal{I}[d] = \mathcal{I}[d']$;

Assuming that d and d' are concepts, and c is a constant.

Entailment

$$S \models \alpha$$

iff for every $\mathcal{I}$

if $\mathcal{I} \models S$

then $\mathcal{I} \models \alpha$

Examples

$$\textbf{AND}\ \texttt{Doctor}\ \texttt{Female}]\ \sqsubseteq\ \texttt{Doctor}])$$

$$\texttt{john}\ \rightarrow\ \texttt{Thing})$$

$$(\texttt{Surgeon}\ \sqsubseteq\ \texttt{Doctor})$$

$$KB \models [(\textbf{AND}\ \texttt{Doctor}\ \texttt{Female}]\ \sqsubseteq\ \texttt{Doctor})$$

$$(\texttt{Surgeon} \doteq [\textbf{AND}\ \texttt{Doctor}\ [\texttt{FILLS} : \texttt{Specialty}\ \texttt{surgery}])$$

Computing Entailments

We want to be able to determine if $KB \models \alpha$, for sentences α of the form:

$$(\texttt{c} \ \rightarrow \ \texttt{d})$$

$$(\texttt{d} \ \subseteq \ \texttt{e})$$

Simplifying the Knowledge Base

- Remove sentences of the form

$$(\texttt{c} \ \rightarrow \ \texttt{d})$$

- Left Hand side of $\subseteq$ and $\doteq$ sentences must be atomic concepts other than `Thing` and each atom appears on the left hand side only once.
- Assume $\subseteq$ and $\doteq$ sentences are acyclic.

Normalization

1. **expand definitions:**

   $(\texttt{Surgeon} \doteq [\textbf{AND}\ \texttt{Doctor}\ [\texttt{FILLS} : \texttt{Specialty}\ \texttt{surgery}]])$

   $$[\textbf{AND} \ldots\ \texttt{Surgeon}\ \ldots]$$

   expands to

   $[\textbf{AND} \ldots\ [\textbf{AND}\ \texttt{Doctor}\ [\texttt{FILLS} : \texttt{Specialty}\ \texttt{surgery}]]\ \ldots]$

2. **flatten the AND operators:**

   $$\textbf{AND}\ \ldots [\texttt{AND}\ d_1\ \ldots\ d_n] \ldots]$$

   can be simlified to:

   $$\textbf{AND}\ \ldots\ d_1\ \ldots\ d_n\ \ldots]$$

Normalization (cont)

3. **combine the ALL operators:**

   $$\textbf{AND} \ \ldots [\textbf{ALL}\, r\ d_1]\ \ldots [\textbf{ALL}\, r\ d_2] \ldots]$$

   can be simlified to:

   $$\textbf{AND} \ \ldots [\textbf{ALL}\, r\ [\textbf{AND}\, d_1\ d_2]] \ldots]$$

4. **combine EXISTS operators:**

   $$\textbf{AND} \ \ldots [\textbf{EXISTS}\, n_1\ r]\ \ldots [\textbf{EXISTS}\, n_2\ r] \ldots]$$

   can be simplified to

   $$\textbf{AND} \ \ldots [\textbf{EXISTS}\, n\ r] \ldots]$$

   where n is the maximum of n_1 and n_2.

Normalization(cont)

5. **deal with Thing:**

   Remove vacuous concepts as arguments to **AND**

   $$[\textbf{ALL}\ r\ \texttt{Thing}]$$

6. **remove redundant expressions:**

The Final Result

$$
\begin{aligned}
&[\mathbf{AND}\ a_1\ \ldots\ a_m \\
&\quad [\mathbf{FILLS}\ r_1\ c_1] \ldots [\mathbf{FILLS}\ r_{m'}\ c_{m'}] \\
&\quad [\mathbf{EXISTS}\ n_1\ s_1] \ldots [\mathbf{EXISTS}\ n_{m''}\ s_{m''}] \\
&\quad [\mathbf{ALL}\ t_1\ e_1] \ldots [\mathbf{ALL}\ t_{m'''}\ e_{m'''}]
\end{aligned}
$$

Structure Mapping

$$KB \models (d \sqsubseteq e)$$

IDEA: For d to be subsumed by e, the normalized d must account for each component of the normalized e in some way

Structure Mapping Procedure

Input: Two normalized concepts d and e where d is of the form [AND $d_1 \; \ldots \; d_m$] and e is of the form [AND $d_1 \; \ldots \; d_m$]

Output yes or no, according to whether $KB \models (d \sqsubseteq e)$

Return yes iff for each component e_j, there exists a component d_i such that d_i matches e_j as follows:

1. if e_j is an atomic concept, then either d_i is identical to e_j, or there is a sentence of the form $(d_i \subseteq d')$ in the KB, where recursively some coponent of d' matches e_j;
2. if e_j is of the form [FILLS r c], then d_i must be identical to it;
3. if e_j is of the form [EXISTS n r], then the corresponding d_i must be of the form [EXISTS n' r], for some $n' \geq n$; if $n = 1$, then d_i may be of the form [FILLS r c];
4. if e_j is of the form [ALL r e'], then d_i must be of the form [ALL r d'], where recursively d' is subsumed by e'.

Computing Satisfaction

$$KB \models (c \rightarrow e)$$

iff

$$KB \models (d \subseteq e)$$

where d is the `AND` of every concept d_i such that $(c \rightarrow d_i)$ is in the KB

Taxonomies and Classification

given some query concept, q, find all c in the KB such that

$$KB \models (c \rightarrow q)$$

given a constant c, find all the atomic concepts a such that

$$KB \models (c \rightarrow a)$$

Partial Order, Classification

Computing Classification

Consider adding a sentence ($a \doteq d$) to a taxonomy.

1. First calculate S, the *most specific subsumers* of d

2. Next calculate G, the *most general subsumees* of d.

3. If there is a concept a' in $S \cap G$, then the concept is already present.

4. Otherwise, insert a.

Computing (cont)

1. Computing most specific subsumers.

2. Computing the most general subsumees.

Classification (cont)

1. Answering Questions
2. Taxonomies and Frame Hierarchies
3. Inheritance