Logic

- ▶ Syntax

    $x \geq y$ **Yes!**

    $x\, y \geq$ **No!**

- ▶ Semantics
    - ▶ $x \geq y$ **FALSE** when y is bigger than x
    - ▶ Under what states of affairs or worlds is $x \geq y$ ture.

Logic

An interpretation **I** *is a mapping between the elements of the* language *and the elements of a conceptualization of the world.*

Logic

- **Entailment**
  $$\textbf{KB} \models \alpha$$

- **Derivability**
  $$\textbf{KB} \vdash_i \alpha$$

  $$\textbf{KB} \vdash \alpha$$

- **Sound**
- **Complete**

Propositional Logic

- **Conjunction** $A \land B$
- **Disjunction** $B \lor C$
- **Implication** $A \rightarrow B$
  premise, antecedent A

  conclusion, consequent B

- **Equivalence** $A \leftrightarrow B$
- **Negation** $\neg B$

Propositional Logic

Syntax

    True, False

    P, Q, . . . Propositional Symbols

    $\wedge, \vee, \leftrightarrow, \rightarrow, \neg, (,)$

    Sentence $\Rightarrow$

                    True

                   |*False*

                   |*PropositionalSymbol*

                   |(*Sentence*)

                   |*Sentence* $\wedge$ *Sentence*

                   |*Sentence* $\vee$ *Sentence*

                   |*Sentence* $\rightarrow$ *Sentence*

                   |*Sentence* $\leftrightarrow$ *Sentence*

                   |$\neg$*Sentence*

Propositional Logic

Semantics

- ► **P** "A is on B"
- ► $I(\mathcal{P}) =$ **True** or $I(\mathcal{P}) =$ **False** for all propositional symbols P.
- ► **True** – true
- ► **False** – false

Propositional Logic

Truth Tables

P	Q	$P \wedge Q$
true	true	true
false	true	false
true	false	false
false	false	false

P	Q	$P \vee Q$
true	true	true
false	true	true
true	false	true
false	false	false

Propositional Logic

Truth Tables

P	Q	$P \rightarrow Q$
true	true	true
false	true	true
true	false	false
false	false	true

P	Q	$P \leftrightarrow Q$
true	true	true
false	true	false
true	false	false
false	false	true

Propositional Logic

Truth Tables

P	$\neg P$
true	false
false	true

Propositional Logic

$$I(P) = T$$
$$I(Q) = F$$
$$I(R) = F$$
$$I(S) = T$$

$$I \models (P \land Q) \lor R$$

Propositional Logic

Semantics

- ▶ Interpretation
- ▶ Compositionality

- ▶ Valid, Tautology
- ▶ Satisfiable
- ▶ Unsatisfiable

Propositional Logic

Truth Tables

P	Q	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q$	$(\neg P \vee Q) \leftrightarrow (P \rightarrow Q)$
true	true	false	true	true	true
false	true	true	true	true	true
true	false	false	false	false	true
false	false	true	true	true	true

Propositional Logic

Rules of Inference

$$\alpha \vdash \beta$$

$$\frac{\alpha}{\beta}$$

Modus Ponens

$$\frac{\alpha \rightarrow \beta, \alpha}{\beta}$$

Rules of Inference

And-Elimination

$$\frac{\alpha_1 \wedge \alpha_2 \ldots \wedge \alpha_n}{\alpha_i}$$

And-Introduction

$$\frac{\alpha_1, \alpha_2, \ldots, \alpha_n}{\alpha_1 \wedge \alpha_2 \ldots \wedge \alpha_n}$$

Rules of Inference

Or-Introduction

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \ldots \vee \alpha_n}$$

Double Negation Elimination

$$\frac{\neg\neg\alpha}{\alpha}$$

Rules of Inference

Unit Resolution

$$\frac{\alpha \vee \beta, \neg \beta}{\alpha}$$

Resolution

$$\frac{\alpha \vee \beta, \neg \beta \vee \gamma}{\alpha \vee \gamma}$$