Learning

Symbolic

Non-Symbolic -- Neural Networks
Symbolic

The approach to learning covered in this segment will be quite a bit different from that of neural networks. The output of the learning procedure will be a symbolic expression that is interpretable and can be combined with other knowledge in symbolic form.

On the other hand the approaches to be covered here are very sensitive to noisy data. The neural network approach is relatively robust in the presence of noise.
The topics to be considered are as follows:

1) Decision Trees
2) Learning Logical Descriptions
   (a) Current best hypothesis
   (b) Version Spaces
Learning Decision Trees

Consider the following data set from Russell and Norvig.

It consists of twelve restaurant visits. They are characterized by 10 different attributes. The goal to be learned is whether or not the people involved in these visits will wait at the restaurant for a table or go elsewhere.
Restaurant Example

(Table From Russell and Norvig)
Inductive Learning

In any inductive learning learning problem we have a set of input output pairs. The output $Y$ is the categorization that we want to learn.

$$(X, Y) \quad X = \text{input} \quad Y = \text{output}$$

$y = f(x)$

From a collection of examples, we want a function $h$ that approximates $f$ as closely as possible.

The aim of the learning problem is to construct $h$
Restaurant Example Continued

Here is $f$ for the example above. It is the decision tree actually used by the customers to generate the behavior found in the restaurant visits given above.

A decision tree for deciding whether to wait for a table
The Algorithm

Here is the first step in constructing $h$ for the data above. The $h$ should give the same behavior as $f$ on the data set – but there is no reason why it should be identical. We have not guarantee that it will give the same results on other examples.
The Algorithm

![Decision Tree Diagram]

Figure 18.6 Splitting the examples by testing on attributes. In (a), we see that Patrons is a good attribute to test first; in (b), we see that Type is a poor one; and in (c), we see that Hungry is a fairly good second test, given that Patrons is the first test.

Note that the decision tree algorithm chooses as
The Algorithm

its next attribute the one that yields the largest number of definitive answers for the examples in the answer set. You should see why Patrons is a better attribute to pick than Type.
Applications

Induction of Rules for: Chemical Process Control
What control settings yield substances of high quality— in the production of pellets of uranium dioxide. (Leech 1986)

Credit Analysis for Loans
American Express UK (Michie 1989)

Diagnosis of Mechanical Devices

Classification of Celestial Objects
Logic and Generalization

Logic as a representation language yields a notion of learning and generalization.

If hypothesis H1 with definition C1 is a generalization of hypothesis H2 with definition C2 then we must have:

$$\forall x C_2(x) \rightarrow C_1(x)$$

The following are ways to generalize a logical expression in accordance with this definition.
1. Replacing Constants with Variables
   Color(Obj1, red)
   Generalizes to
   Color(X, red)
   Color(obj, red) => Color(Obj, X)

2) Adding a disjunct to an expression

   Shape(X, round) \ Size(X, small)
   \ Color(X, red)
   generalizes to
   Shape(X, round) \ Size(X, small) \ (Color(X, red) \ Color(x, blue))
3. Dropping conditions from a conjunctive expression:

\[ \text{Color}(x, \text{red}) \land \text{Shape}(X, \text{round}) \land \text{Size}(x, \text{small}) \]

generalizes to

\[ \text{Color}(x, \text{red}) \land \text{Shape}(x, \text{round}) \]

4) Replacing a property with its parent in a class hierarchy.

If we know that primary-Color is a superclass of Color, then

\[ \text{Color}(x, \text{red}) \]

generalizes to

\[ \text{Color}(x, \text{primary-color}) \]
Current Best Hypothesis Search

The idea here is to just keep the best hypothesis. So, one begins with a positive example and constructs an expression that covers the positive example and as little else as possible.

Then when a negative example is considered, the hypothesis is specialized to exclude the negative example — but yet to still include all positive examples considered so far.

When a positive example is considered, the hypothesis is generalized to cover this positive example, but yet to still not cover any of the negative examples considered so far.
Examples

Example X1 -- Positive

H1: \( \forall x \text{ WillWait}(x) \iff \text{Alternate} \)

Example X2 -- Negative

H2: \( \forall x \text{ WillWait}(x) \iff \)
\( \text{Alternate}(x) \land \text{Patrons}(x, s) \)
Examples (cont)

Example X3 -- Positive

H3: \( \forall x \) WillWait(x) --\( \rightarrow \) Patrons(x, Some)

Example X4 -- Positive

H4: \( \forall x \) WillWait(x) \&lt;\( \rightarrow \) Patrons(x, some) \( \rightarrow \)
\( \rightarrow \)
(Patrons(x, Full) \& Fri/Sat(x))
Version Spaces

The current-best hypothesis search approach only considers one hypothesis at a time – even though at each point there may be multiple hypotheses that can be entertained. Thus backtracking is necessary.

The version space algorithm represents the space of all possible hypotheses that are consistent with the data seen so far. This is the set of hypotheses between G (most general boundary) and S (most specific boundary).
Version Spaces

Converging boundaries of the G and S Sets in the candidate elimination algorithm.
Version Spaces (cont)

G – General Boundary Set S – Most Specific Boundary Set

The current version space is the set of hypotheses consistent with the examples so far.

Every member of the S – Set is consistent with obs so far, and there are no consistent hypothesis that are more specific.

Every member of the G – Set is Consistent with obs so far, and there are no consistent hypotheses that are more general.
example

The set of hypotheses is viewed as a lattice with the most general on the top and the most specific at the bottom. This is illustrated below for a simple language characterizing objects as for size, color, and shape. (example taken from Luger and Stubblefield).
example

G: True (most general Hypotheses)
S: False (empty)

Initially we set G to be the most general hypothesis and S to be the most specific hypothesis.
Example (cont)

First consider separately half of the version space algorithm. This half considers only positive examples, begins with the most specific \( S \) and generalizes to cover each example in turn.

Specific to general search of the version space learning the concept "ball"
Example (cont)

Now consider the other half – beginning with the most general G and specializing G to exclude negative examples, and then eliminating those Gs that do not cover positive examples.

Note our language has the following sizes, colors, and shapes:
Example (cont)

small, large
red, white, blue
cubes, bricks, balls
Candidate Elimination Algorithm

Now we put both halves of the algorithm together and modify both S and G as examples are considered.

Initialize G to contain one element,
   the null description (all variables)
Initialize S to contain one element:
   the first positive example.

i) For each new positive instance p:
   Delete all members of
       G that fail to match P.
   For every s in S,
       if s does not match P, replace s with
           its most specific generalizations
               that match P.
   Delete from S any hypothesis more general
      than some other hypothesis in S
   Delete from S any hypothesis more general
      than some hypothesis in G
Candidate Elimination Algorithm (cont)

ii) For each new negative instance n:
   Delete all members of s that match n.
   For each g in G that matches n,
   replace g with its most general
   specializations that do not match n.
   Delete from G any hypothesis more
   specific than some
   other hypothesis in G.
   Delete from G any hypothesis more
   specific than some
   hypothesis in S.

iii) If G = S and both are singletons
    Concept found
    If G, S become empty,
    Fail
Final Illustration

Now here is the algorithm put together on the same example.

G: \{obj(X, Y, Z)\}
S: {}

Positive: obj(small, red, ball)

G: \{obj(X, Y, Z)\}
S: \{obj(small, red, ball)\}

Negative: obj(small, blue, brick)

G: \{obj(X, red, Z), obj(X, Y, ball)\}
S: \{obj(small, red, ball)\}

Positive: obj(large, red, ball)

G: \{obj(X, red, Z), obj(X, Y, ball)\}
S: \{obj(X, red, ball)\}

Negative: obj(large, red, cube)

G: \{obj(X, Y, ball)\}
S: \{obj(X, red, ball)\}

The candidate elimination algorithm learning the concept "red ball."