A Two-Step Transition to Higher Mathematics

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My First Calculus Course

2-14-90

Math 150A - Spring 1990 - Midterm 1 - Dr. Martelli

23

1. (3) Give the geometric definition of

$$\lim_{x \to x_0} f(x) = L$$

2. (a) (3) Prove the following result

Theorem on the Uniqueness of the Limit

Let

$$\lim_{x \to x_0} f(x) = L \quad \text{and} \quad \lim_{x \to x_0} f(x) = M$$

Then L=M.

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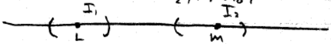
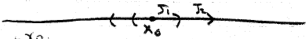
2) a) Proof.

Assume $L \neq M$, and $L < M$. Choose an open interval I_1 centered at L and an open interval I_2 centered at M which do not overlap. From the given information, we know there must exist an open interval J_1 centered at x_0 such that for all $x \in J_1$, $x \neq x_0$, we have $f(x) \in I_1$. ~~We~~ Also ~~assume~~ there exists an open interval J_2 centered at x_0 such that for all $x \in J_2$, $x \neq x_0$, we have $f(x) \in I_2$.





Pick a point x ($x \neq x_0$) in both J_1 and J_2. Then $f(x)$ must be in both I_1 and I_2. This is impossible since I_1 and I_2 are disjoint. Therefore $L \neq M$ cannot be accepted. Therefore $L = M$.

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1. (4) State and prove the Intermediate Value Theorem

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5. The following propositions describing certain properties of the function f are false. Find the
ones which are true.

(2) P_1 : for every pair of points x,y with x<y , we have $f(x) < f(y)$

(1) P_2: $f(x) = 0$ for all $x > 0$.

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Math 150A - Midterm # 4 - Spring 1990 - Dr. Martelli 2

1.Let f:[a,b]→R be continuous on [a,b] and differentiable on (a,b).

(a) (2.5) Assume that f '(x) ≥ 0 for all x∈(a,b). Prove that f is increasing on [a,b].

(b) (2.5) Assume, moreover that f '(x) = 0 only finitely many times on (a,b). Prove that f is strictly increasing on [a,b].

$f'(x) = (x^2+2)[-1](x^2+1)^{-2}(2x) + (2x)(x^2$

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1) a) let x, y be any pair of points in (a,b). Assume $x<y$. By the MVT, there exists a point $c \in (x,y)$ such that $f'(c) = \frac{f(y)-f(x)}{y-x}$, hence $f(y)-f(x) = f'(c)(y-x)$. Since $f'(c) \geq 0$, we have $f(y) \geq f(x)$. Thus f is increasing.

Excellent!

b) We want to show that f is strictly increasing on $[a,b]$. The function would fail to be strictly increasing if we can find two points, $x<y$, such that $f(x)=f(y)$. Since f is increasing, it must be constant on (x,y). This, however, would force $f'(x)$ to be zero an infinate number of times. That is impossible, hence f is strictly increasing.

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- Reading and recreating proofs.
- Regular use of the symbols and language of foundational mathematics.

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- Step 2: Creative theorem proving and analysis

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- Course materials consist of a set of notes developed by MU faculty, as well as a secondary reference text.

Course Topics

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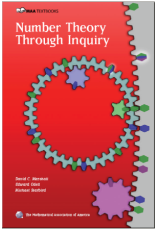
- MA 314 is designed to introduce students to a mathematical perspective that features active participation in developing ideas that are new to the students and in developing proofs of mathematical assertions.

Number Theory Through Inquiry

Number Theory Through Inquiry by Marshall, Odell, and Starbird, MAA TEXTBOOKS, 2007.



2008 PREP

2008 MAA PREP Workshop, Inquiry Based Learning with a Focus on Number Theory: A Transitions to Proof Course

The workshop will introduce participants to the IBL style of instruction and will specifically show them how to teach a transitions-to-proof number theory course in that style. Participants will be connected to a mentoring support system to help them as they implement these ideas in their own institutions. Participants should be fully prepared to teach their own IBL-style courses after the workshop, particularly number theory.

Effectiveness

- *A sense-making approach to proof: Strategies of students in traditional and problem-based number theory courses*
- Jennifer Christian Smith
- Journal of Mathematical Behavior, **25**, 2006, 73-90

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In what ways do the conceptions of and aproaches to constructing and validating proofs of students in a MMM course differ from those of students in a lecture-based course?

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- approached the construction of proofs in ways that demonstrated efforts to make sense of the mathematical ideas;
- employed this sense making approach when validating mathematical proofs.

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- Consider an inquiry based transition-to-proofs course.